(Revised course)

(3 Hours) [Total Marks : 80]

N.B. : (1) Question No. **1** is **compulsory**.

(2) Answer any **three** questions from question nos. 2 to 6.

(3) **Figures** to the **right** indicate **full** marks.

(4) Programming Calculators are **not allowed**.

1. (a) Evaluate $\int_0^\infty x^2 \, 7^{-4x^2} \, dx$ 3

    (b) Solve $(D^4+4)y = 0$ 3

    (c) Prove that $E\nabla = \Delta = \nabla E$ 3

    (d) Solve $(x + 2y^3) \dfrac{dy}{dx} = y$.   3

    (e) Evaluate $\displaystyle\iint_R r^3 \, dr d\theta$ over the region between the circles $r = 2\sin\theta$, $r = 4\sin\theta$.   4

    (f) Evaluate $\displaystyle\int_0^1 \int_y^{\sqrt{y}} \dfrac{x}{(1-y)\sqrt{y-x^2}} \, dy dx$ 4

2. (a) Solve :- $(x^3y^4 + x^2y^3 + xy^2 + y) \, dx + (x^4y^3 - x^3y^2 - x^2y + x) dy = 0$ 6

    (b) Change the order of integral and hence evaluate $\displaystyle\int_0^5 \int_{2-x}^{2+x} dx dy$ 6

    (c) Prove that $\displaystyle\int_0^1 \dfrac{x^2}{\sqrt{1-x^4}} dx \int_0^1 \dfrac{dx}{\sqrt{1+x^4}} = \dfrac{\pi}{4\sqrt{2}}$ 8

3. (a) Evaluate $\displaystyle\int_0^1 \int_{y^2}^1 \int_0^{1-x} x \, dx dy dz$.   6

    (b) Find the area of one loop of the lemniscate $r^2 = a^2 . \cos 2\theta$ 6

    (c) Solve $(D^3 + 2D^2 + D)y = x^2 e^{3x} + \sin^2 x + 2^x$.   8

4. (a) Show that the length of arc of the parabola $y^2 = 4ax$ cut off by the line $3y = 8x$ is   6

      $a\left(\log 2 + \dfrac{15}{16}\right)$

    (b) Using the method of variation of parameters solve $\dfrac{d^2y}{dx^2} + 4y = \tan 2x$.   6

    (c) Compute $y(0.2)$ given $\dfrac{dy}{dx} + y + xy^2 = 0$, $y(0) = 1$ by taking $h = 0.1$ using Runge-Kutta   8

      method of fourth order correct to 4 decimals.

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5.   (a)   Solve $\dfrac{dy}{dx} + x(x+y) = x^3(x+y)^3 - 1$.   **6**

     (b)   Solve $\dfrac{dy}{dx} - 2y = 3e^x$, $y(0) = 0$ using Taylor series method. Find approximate value   **6**

         of y for x = 1 and 1.1.

     (c)   Evaluate $\displaystyle\int_0^6 \dfrac{dx}{1+x}$ using   **8**

         (i)   Trapezoidal rule

         (ii)   Simpson's $\left(\dfrac{1}{3}\right)^{rd}$ rule and

         (iii)   Simpson's $\left(\dfrac{3}{8}\right)^{th}$ rule.

         Compare result with exact values.

6.   (a)   The current in a circuit containing an inductance L, registance R and voltage   **6**

         E sin wt is given by

$$L\dfrac{di}{dt} + Ri = E\sin wt$$

         If i = 0 at t = 0, find i.

     (b)   Evaluate $\displaystyle\iint_R e^{2x-3y}\, dx\,dy$ over the triangle bounded by x + y = 1, x = 1, y = 1.   **6**

     (c)   (i)   Find the volume of solid bounded by the surfaces $y^2 = 4ax$, $x^2 = 4ay$ and the   **4**

             planes Z = 0, Z = 3.

         (ii)   Change to polar co-ordinates and evaluate   **4**

$$\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \dfrac{dx\,dy}{\sqrt{a^2 - x^2 - y^2}}$$